CQC
Composing Quantum Channels

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04.03.2014
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Every process can be regarded as a channel
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\[ \rho \rightarrow T \rightarrow \rho' = T(\rho) \]
Motivation

Every process can be regarded as a channel

Quantum channel:
\( \rho \rightarrow T \rightarrow \rho' = T(\rho) \)

Sequential composition of quantum channels:
\( \rho \rightarrow T \rightarrow \rho' \rightarrow T \rightarrow \rho'' \rightarrow \ldots \rightarrow T \rightarrow T^t(\rho) \)

Parallel composition of quantum channels:
\( \rho_n \rightarrow T \rightarrow \ldots \rightarrow T \otimes_n (\rho_n) \)
Motivation

Every process can be regarded as a channel

The CQC project has as purpose studying quantum channels with emphasis on the behaviour under composition.
Goals
Goals

01: Theory and tools
Goals

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O1: Theory and tools

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- O1: Theory and tools
- O2: Implications
- O3: Dissemination
Workpackages

WP1: Single Channels

WP2: Sequential Composition

O1: Theory and tools

O2: Implications

O3: Dissemination

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Workpackages

WP1: Single Channels
WP2: Sequential Composition
WP3: Parallel Composition
O1: Theory and tools
O2: Implications
O3: Dissemination
Workpackages

WP1: Single Channels

WP3: Parallel Composition

WP2: Sequential Composition

WP4: Complexity Theory

O1: Theory and tools

O2: Implications

O3: Dissemination
Workpackages

WP1: Single Channels
WP2: Sequential Composition
WP3: Parallel Composition
WP4: Complexity Theory
WP5: Dissemination

O1: Theory and tools
O2: Implications
O3: Dissemination
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Team

Prof. M. Wolf
Motohisa Fukuda
Daniel Reitzner
Alexander Müller-Hermes

Collaboration: 6 joint publications.
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Capacity of Classical Channels

The capacity of a classical channel can be operationally defined as:

\[ C_c(\mathcal{N}) = \frac{\text{\# of transmitted bits with error } \epsilon \rightarrow 0}{\text{\# of required parallel uses of the channel}} \]

**Theorem (Shannon)**

\[ C_c(\mathcal{N}) = \max_{P_x} S(X) - S(X|Y) \]
Classical Capacity of Quantum Channels

The classical capacity of a quantum channel can be defined as:

\[ C^1(\mathcal{N})_{\text{reg}} = \lim_{\epsilon \to 0} \limsup_{k \to \infty} \left( \frac{m}{k} : \exists A, B, \| \text{Id}_{2^m} - B \circ \mathcal{N}^\otimes k \circ A \| < \epsilon \right) \]
Classical Capacity of Quantum Channels

1) Capacity can be increased with assisted entanglement
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2) Object of study: the $d$-restricted classical capacity of a quantum channel, $C^d(\mathcal{N})_{\text{reg}}$. 
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3) If $d = 1$ we recover the classical capacity
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4) if $d = n$, the input dimension of the channel, we recover the entanglement-assisted classical capacity
Classical Capacity of Quantum Channels

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3) If $d = 1$ we recover the classical capacity
4) If $d = n$, the input dimension of the channel, we recover the entanglement-assisted classical capacity

**Theorem (arXiv 1305.1020)**

Given a channel $\mathcal{N}$, its product input state $d$-restricted capacity is given by:

$$C^d(\mathcal{N}) = \sup \left\{ S \left( \sum_{i=1}^{N} \lambda_i (\mathcal{N} \circ \phi_i)((tr_A \otimes I_d)(\eta_i)) \right) \right.$$

$$+ \sum_{i=1}^{N} \lambda_i \left[ S \left( (I_d \otimes tr_B)(\eta_i) \right) - S \left( (I_d \otimes (\mathcal{N} \circ \phi_i))(\eta_i) \right) \right] \right\},$$

where the supremum runs over all $N \in \mathbb{N}$, all probability distributions $(\lambda_i)_{i=1}^{N}$, and all local operations $(\phi_i)_{i=1}^{N}$, and bipartite $d$-dimensional pure states $(\eta_i)_{i=1}^{N}$. 
The need for regularization

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2) The true capacity corresponds to

\[ C^d(N)_{\text{reg}} = \sup_k \frac{C^{dk}(N \otimes k)}{k} \]
The need for regularization

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3) Regularization is needed (Hastings)
The need for regularization

1) The theorem gives the product capacity
2) The true capacity corresponds to

\[ C^d(\mathcal{N})_{\text{reg}} = \sup_k \frac{C^{d^k}(\mathcal{N}^\otimes k)}{k} \]

3) Regularization is needed (Hastings)

**Theorem (arXiv 1305.1020)**

*There exists a family of channels with input dimension 2n such that*

\[ C^{\sqrt{n}}(\mathcal{N}) + t_1 \log n + t_2 \leq \frac{C^n(\mathcal{N}^\otimes 2)}{2} \]
Additivity violation

Entangled input can increase the capacity.
Additivity violation

Entangled input can increase the capacity.
Additivity violation

Entangled input can increase the capacity.

**Theorem (Hastings)**

*There exist some channels $\Phi$ and*

\[ S_{\text{min}}(\Phi \otimes \bar{\Phi}) < S_{\text{min}}(\Phi) + S_{\text{min}}(\bar{\Phi}). \]
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(1) $S_{\text{min}}(\cdot)$ is defined by

$$S_{\text{min}}(\Phi) = \min_{\rho} S((\Phi(\rho))$$

where $S(\cdot)$ is the von Neumann entropy.
Additivity violation

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(2) \( \bar{\Phi} \) is the complex conjugate of \( \Phi \).
Additivity violation

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where \(S(\cdot)\) is the von Neumann entropy.

(2) \(\bar{\Phi}\) is the complex conjugate of \(\Phi\).

(3) The intuition behind is

\[
U \otimes \bar{U} |b\rangle = |b\rangle
\]

where \(|b\rangle\) is a Bell state.
Improvement

The additivity violation can be proved by Lévy’s Lemma and $\varepsilon$-net method when $l \sim n \gg k^2$ where $l$, $n$, $k$ are dimensions of input, output and environment spaces.

Previously, $l \sim n$ case was proven by Aubrun, Szarek and Werner (ASW), but with some stronger constraints on $l_k^2$ and $n_k^2$, which we do not need.

The same method was used by Hayden, Leung and Winter to show that typical subspaces are highly entangled. Their technique was improved by our new trick.
Improvement

**Theorem (Fukuda)**

The additivity violation can be proved by Lévy’s Lemma and $\epsilon$-net method when $l \sim n \gtrsim k^2$ where $l, n, k$ are dimensions of input, output and environment spaces.
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Is the pair additive?

We know that typically

\[ S_{\text{min}}(\Phi \otimes \bar{\Phi}) < S_{\text{min}}(\Phi) + S_{\text{min}}(\bar{\Phi}) \]
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But it still may be, that

\[ S_{\text{min}}((\Phi \otimes \bar{\Phi})^{\otimes n}) \neq n \cdot S_{\text{min}}(\Phi \otimes \bar{\Phi}) \]

because entangled inputs may work well only for the pair \( \Phi \otimes \bar{\Phi} \) and may not go beyond.
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**Theorem (Fukuda, Nechita)**

*If you fix an input state and take random channels, then products of Bell states minimize the output entropy for \((\Phi \otimes \bar{\Phi})^{\otimes n}\).*
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**Theorem (Fukuda, Nechita)**

*If you fix an input state and take random channels, then products of Bell states minimize the output entropy for $(\Phi \otimes \bar{\Phi})^\otimes n$.*

This could be stepping stones for the above conjecture.
What is the optimal rate of information storage in a quantum memory?

Noise channel $\mathcal{T}_t: \mathcal{M}_d \to \mathcal{M}_d$ is Markovian, i.e. $\mathcal{T}_t = e^{tL}$. 
What is the optimal rate of information storage in a quantum memory?

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\[ DE \otimes_{mm} T^\otimes_{mm} \]
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Diagram:

- $E$
- $C_1$
- $C_2$
- $C_3$
- $D$

$T_{t/k}^\otimes$
What is the optimal rate of information storage in a quantum memory?

Noise channel $\mathcal{T}_t : \mathcal{M}_d \rightarrow \mathcal{M}_d$ is Markovian, i.e. $\mathcal{T}_t = e^{t\mathcal{L}}$.

Quantum subdivision capacity: $Q_c(t\mathcal{L})$
What is the optimal rate of information storage in a quantum memory?

Noise channel $\mathcal{T}_t : \mathcal{M}_d \rightarrow \mathcal{M}_d$ is Markovian, i.e. $\mathcal{T}_t = e^{t\mathcal{L}}$.

Quantum subdivision capacity: $Q_C(t\mathcal{L})$

Different choices of $C$ lead to different capacities!
Definition (MH, Reeb, Wolf 2013)

The \textit{quantum subdivision capacity} of $t\mathcal{L}$ is then defined as the supremum of asymptotical achievable rates

$$Q_{\mathcal{C}}(t\mathcal{L}) := \sup\{ R \in \mathbb{R}^+ : R = \limsup_{\nu \to \infty} \frac{n_\nu}{m_\nu} \}.$$  

such that the asymptotic communication error vanishes

$$\inf_{k, \mathcal{E}, \mathcal{D}, \mathcal{C}_1, \ldots, \mathcal{C}_k} \left\| \text{id}_{2^{n_\nu}} \otimes - \mathcal{D} \circ \prod_{l=1}^{k} \left( \mathcal{C}_l \circ \left( e_{\nu}^{\frac{t}{2}} \mathcal{L} \right)^\otimes_{m_\nu} \right) \circ \mathcal{E} \right\|_\diamond \to 0 \quad \text{as} \quad \nu \to \infty.$$  

Infimum goes over:

- $k \in \mathbb{N}$ number of subdivisions
- $\mathcal{E} : \mathcal{M}_{2^{n_\nu}} \to \mathcal{M}_{d^{m_\nu}}$ and $\mathcal{D} : \mathcal{M}_{d^{m_\nu}} \to \mathcal{M}_{2^{n_\nu}}$ quantum channels
- $\mathcal{C}_l \in \mathcal{C}$ channels from the subset $\mathcal{C}$
Infinitely divisible coding maps

1. Example: Let $\mathcal{C}$ be the set of infinitely divisible quantum channels
Infinitely divisible coding maps

1. Example: Let $\mathcal{C}$ be the set of infinitely divisible quantum channels

- $\mathcal{C} = \prod_{i=1}^{N} e^{\mathcal{L}_i^l}$ for some coding Liouvillians $\mathcal{L}_i^l$
Infinitely divisible coding maps

1. Example: Let $\mathcal{C}$ be the set of infinitely divisible quantum channels
   
   \[ C_i = \prod_{i=1}^{N} e^{L_i^1} \text{ for some coding Liouvillians } L_i \]

   ![Diagram of coding channels](image)
Infinitely divisible coding maps

1. Example: Let $\mathcal{C}$ be the set of infinitely divisible quantum channels

   $\mathcal{C} = \prod_{i=1}^{N} e^{\mathcal{L}_i}$ for some coding Liouvillians $\mathcal{L}_i$

![Diagram](image_url)

Theorem (MH, Reeb, Wolf 2013)

For any noise Liouvillian $\mathcal{L} : \mathcal{M}_d \rightarrow \mathcal{M}_d$ and any $t \in \mathbb{R}^+$ we have

$$Q_{ID}(t \mathcal{L}) = \log(d)$$
Unitary coding maps

2. Example: \( \mathcal{E} = \mathcal{U} \), i.e. unitary coding maps.
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\[ E \rightarrow U_1 \rightarrow U_2 \rightarrow U_3 \rightarrow D \]

\[ T_{t/k} \otimes m \otimes m \otimes m \otimes m \]

Theorem (MH, Reeb, Wolf 2013)

For any noise Liouvillian \( 
\begin{align*}
L & : \mathcal{M}_d \rightarrow \mathcal{M}_d \\
\end{align*}
\)
and any \( t \in \mathbb{R}^+ \) we have

\[ Q_{\mathcal{U}}(tL) > 0 \]
Unitary coding maps

2. Example: \( \mathcal{C} = \mathcal{U} \), i.e. unitary coding maps.

![Diagram of unitary coding maps](image)

**Theorem (MH, Reeb, Wolf 2013)**

For any noise Liouvillian \( \mathcal{L} : \mathcal{M}_d \to \mathcal{M}_d \) and any \( t \in \mathbb{R}^+ \) we have

\[
Q_\mathcal{U}(t\mathcal{L}) > 0
\]
Is $Q_{UL}(tL)$ also $\log(d)$?
Is $Q_{\mathcal{U}}(t\mathcal{L})$ also $\log(d)$?

Answer: No!
Is $Q_{U}(tL)$ also $\log(d)$?

**Answer:** No!

Liouvillian depolarizing onto state $\rho_0 \in \mathcal{M}_d$:

$$L^{\text{dep}}(\rho) := \text{tr}(\rho)\rho_0 - \rho$$
Is $Q_{UL}(t\mathcal{L})$ also $\log(d)$?

Answer: No!

Liouvillian depolarizing onto state $\rho_0 \in \mathcal{M}_d$:

$$\mathcal{L}^{\text{dep}}(\rho) := \text{tr}(\rho)\rho_0 - \rho$$

\[\Leftrightarrow\] Generates depolarizing channel:

$$e^{t\mathcal{L}^{\text{dep}}}(\rho) = (1 - e^{-t})\text{tr}(\rho)\rho_0 + e^{-t}\rho$$
Is $Q_{UL}(tL)$ also $\log(d)$?

Answer: No!

Liouvillian depolarizing onto state $\rho_0 \in \mathcal{M}_d$:

$$L^{\text{dep}}(\rho) := \text{tr}(\rho) \rho_0 - \rho$$

$\rightsquigarrow$ Generates depolarizing channel:

$$e^{tL^{\text{dep}}}(\rho) = (1 - e^{-t}) \text{tr}(\rho) \rho_0 + e^{-t} \rho$$

Theorem (MH, Reeb, Wolf 2013)

For the noise Liouvillian $L^{\text{dep}} : \mathcal{M}_d \to \mathcal{M}_d$ and $t \in \mathbb{R}^+$ we have

$$Q_{UL}(tL^{\text{dep}}) \leq \log(d) - (1 - e^{-t}) S(\rho_0)$$
Definition (MH, Reeb, Wolf 2013)

The $\mathcal{C}$—**quantum subdivision capacity** of noise Liouvillian $t\mathcal{L}$ is then defined as the supremum of asymptotical achievable rates

$$Q_{\mathcal{C}}(t\mathcal{L}) := \sup\{R \in \mathbb{R}^+ : R = \limsup_{\nu \to \infty} \frac{n_\nu}{m_\nu}\}.$$  

such that the asymptotic communication error vanishes

$$\inf_{k, \mathcal{E}, \mathcal{D}, C_1, \ldots, C_k} \left\| \mathrm{id}_{2^n} \otimes n_\nu - \mathcal{D} \circ \prod_{l=1}^{k} \left( C_l \circ \left( e^{t_L} \right)^{\otimes m_\nu} \right) \circ \mathcal{E} \right\|_\diamond \to 0 .$$

as $\nu \to \infty$. Infimum goes over:

- $k \in \mathbb{N}$ number of subdivisions
- $\mathcal{E}: \mathcal{M}_2^{\otimes n_\nu} \to \mathcal{M}_d^{\otimes m_\nu}$ and $\mathcal{D}: \mathcal{M}_d^{\otimes m_\nu} \to \mathcal{M}_2^{\otimes n_\nu}$ quantum channels
- $C_l \in \mathcal{C}$ channels from the subset $\mathcal{C}$
Continuous quantum capacity

Definition (MH, Reeb, Wolf 2013)

The $\mathcal{C}$—continuous quantum capacity of noise Liouvillian $t\mathcal{L}$ is then defined as the supremum of asymptotical achievable rates

$$Q^\text{cont} (t\mathcal{L}) := \sup\{ R \in \mathbb{R}^+ : R = \limsup_{\nu \to \infty} \frac{n_\nu}{m_\nu} \}.$$

such that the asymptotic communication error vanishes

$$\inf_{k, \mathcal{E}, \mathcal{D}, \mathcal{C}_1, \ldots, \mathcal{C}_k} \left\| \text{id}_{2^{n_\nu}} \otimes n_\nu - \mathcal{D} \circ \prod_{l=1}^{k} \left( \mathcal{C}_l \circ \left( e^{t_\nu \mathcal{L}} \right) \otimes m_\nu \right) \circ \mathcal{E} \right\| \to 0.$$

as $\nu \to \infty$. Infimum goes over:

- $k \in \mathbb{N}$ number of subdivisions
- $\mathcal{E} : \mathcal{M}_{2^{n_\nu}} \to \mathcal{M}_{d^{m_\nu}}$ and $\mathcal{D} : \mathcal{M}_{d^{m_\nu}} \to \mathcal{M}_{2^{n_\nu}}$ quantum channels
- $\mathcal{C}_l \in \mathcal{C}$ channels from the subset $\mathcal{C}$
Continuous quantum capacity

**Definition (MH, Reeb, Wolf 2013)**

The $C-$continuous quantum capacity of noise Liouvillian $t\mathcal{L}$ is then defined as the supremum of asymptotical achievable rates

$$Q_c^{\text{cont}}(t\mathcal{L}) := \sup \{ R \in \mathbb{R}^+ : R = \limsup_{\nu \to \infty} \frac{n_\nu}{m_\nu} \}.$$  

such that the asymptotic communication error vanishes

$$\inf_{\mathcal{E}, \mathcal{D}, \mathcal{L}_c} \left\| \text{id}_{2^\otimes n_\nu} - \mathcal{D} \circ T \exp \left( \int_0^t \mathcal{L}^{\otimes m_\nu} + \mathcal{L}_c(t') \, dt' \right) \circ \mathcal{E} \right\| \to 0 \quad .$$

as $\nu \to \infty$. Infimum goes over:

- $\mathcal{E} : M_2^{\otimes n_\nu} \to M_d^{\otimes m_\nu}$ and $\mathcal{D} : M_d^{\otimes m_\nu} \to M_2^{\otimes n_\nu}$ quantum channels
- $\mathcal{L}_c \in \mathcal{C}$ time-dependent coding Liouvillians from the subset $\mathcal{C}$
Can dissipation improve quantum capacity?

Answer: Yes

Even for usual quantum capacity we have:

Theorem (MH, Reeb, Wolf 2013)

There exist time-independent Liouvillians $L: \mathcal{M}_{d} \rightarrow \mathcal{M}_{d}$ and $L': \mathcal{M}_{d} \rightarrow \mathcal{M}_{d}$ where $L'$ is purely dissipative such that

$$Q(e^{L}) < Q(e^{L} + L').$$
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Even for usual quantum capacity we have:

**Theorem (MH, Reeb, Wolf 2013)**

There exist time-independent Liouvillians $\mathcal{L}: \mathcal{M}_d \to \mathcal{M}_d$ and $\mathcal{L}': \mathcal{M}_d \to \mathcal{M}_d$ where $\mathcal{L}'$ is purely dissipative such that

$$Q(e^{\mathcal{L}}) < Q(e^{\mathcal{L} + \mathcal{L}'}).$$
Thank you for your attention.

For more information regarding CQC and a list of publications associated with this project please visit:

http://www-m5.ma.tum.de/CQC