

CQC

Composing Quantum Channels

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Motivation

Every process can be regarded as a channel

Motivation

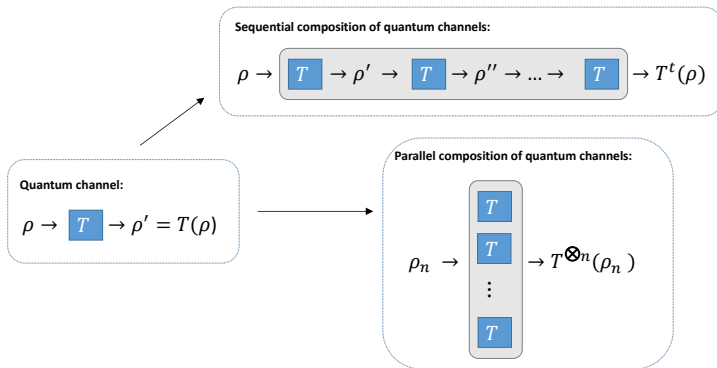
Every process can be regarded as a channel

Quantum channel:

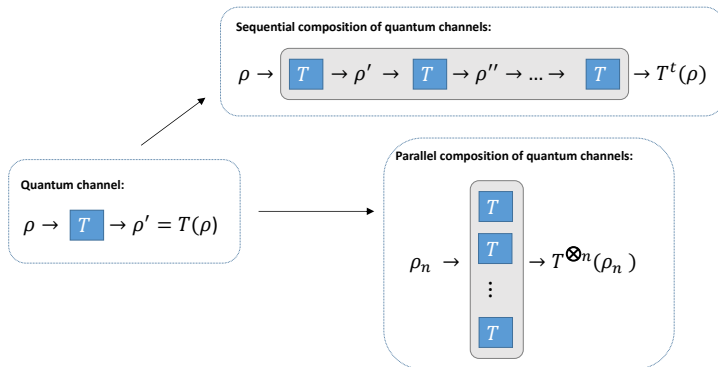
$$\rho \rightarrow \boxed{T} \rightarrow \rho' = T(\rho)$$

Motivation

Every process can be regarded as a channel



Every process can be regarded as a channel



The CQC project has as purpose studying quantum channels with emphasis on the behaviour under composition.

Goals

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O1: Theory and tools

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O1: Theory and tools

O2: Implications

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O1: Theory and tools

O2: Implications

O3: Dissemination

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O1: Theory and tools

O2: Implications

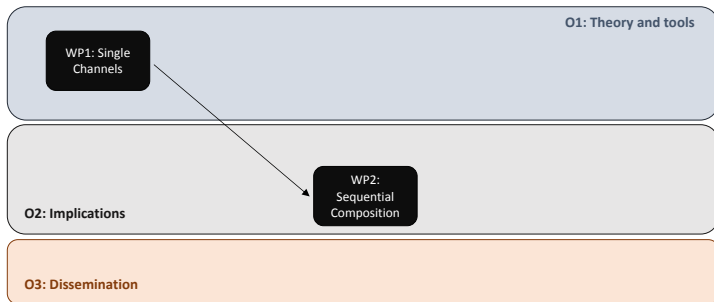
O3: Dissemination

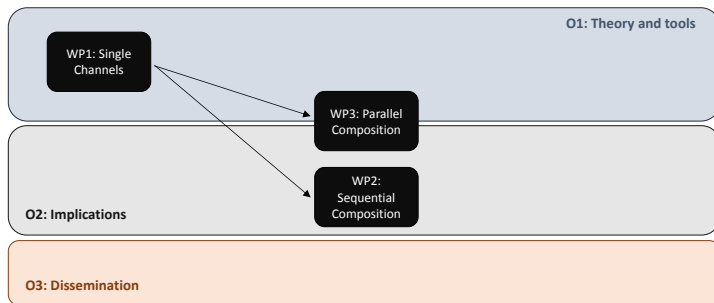
WP1: Single
Channels

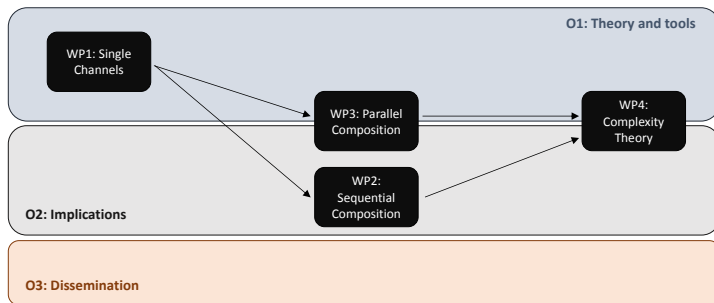
O1: Theory and tools

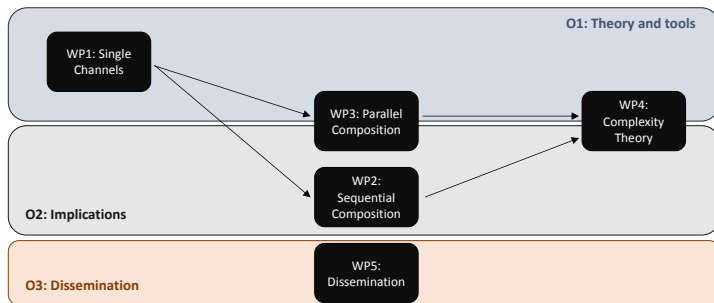
O2: Implications

O3: Dissemination











Prof. M. Wolf

Motohisa Fukuda
Daniel Reitzner
Alexander Müller-Hermes



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MADRID

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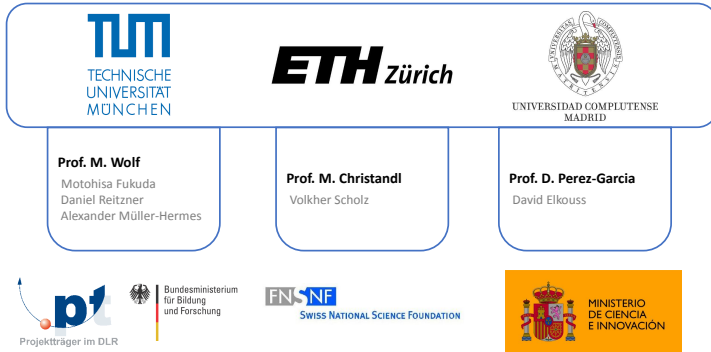
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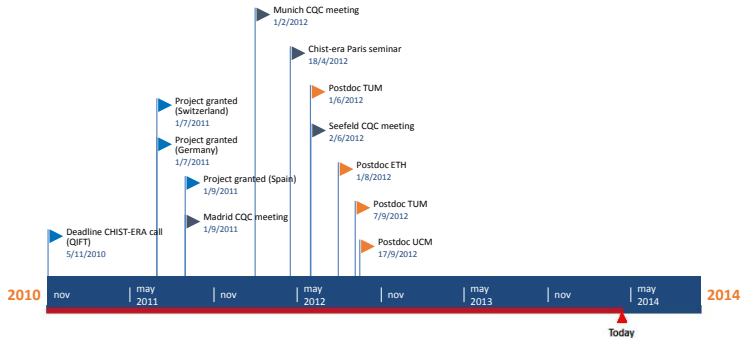
David Elkouss





Collaboration: 6 joint publications.

Timeline



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Capacity of Classical Channels

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The capacity of a classical channel can be operationally defined as:

$$\mathcal{C}_c(\mathcal{N}) = \frac{\# \text{ of transmitted bits with error } \epsilon \rightarrow 0}{\# \text{ of required parallel uses of the channel}}$$

Theorem (Shannon)

$$\mathcal{C}_c(\mathcal{N}) = \max_{P_x} S(X) - S(X|Y)$$

Classical Capacity of Quantum Channels

The classical capacity of a quantum channel can be defined as:

$$\mathcal{C}^1(\mathcal{N})_{\text{reg}} = \lim_{\epsilon \rightarrow 0} \limsup_{k \rightarrow \infty} \left(\frac{m}{k} : \exists A, B, \|\text{Id}_{2^m} - B \circ \mathcal{N}^{\otimes k} \circ A\| < \epsilon \right)$$

Classical Capacity of Quantum Channels

1) Capacity can be increased with assisted entanglement

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Classical Capacity of Quantum Channels

- 1) Capacity can be increased with assisted entanglement
- 2) Object of study: the d -restricted classical capacity of a quantum channel, $\mathcal{C}^d(\mathcal{N})_{\text{reg}}$.

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- 4) if $d = n$, the input dimension of the channel, we recover the entanglement-assisted classical capacity

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- 4) if $d = n$, the input dimension of the channel, we recover the entanglement-assisted classical capacity

Theorem (arXiv 1305.1020)

Given a channel $\mathcal{N}$, its product input state d -restricted capacity is given by:

$$\mathcal{C}^d(\mathcal{N}) = \sup \left\{ S \left(\sum_{i=1}^N \lambda_i (\mathcal{N} \circ \phi_i) ((\text{tr}_A \otimes I_d)(\eta_i)) \right) + \sum_{i=1}^N \lambda_i \left[S \left((I_d \otimes \text{tr}_B)(\eta_i) \right) - S \left((I_d \otimes (\mathcal{N} \circ \phi_i))(\eta_i) \right) \right] \right\},$$

where the supremum runs over all $N \in \mathbb{N}$, all probability distributions $(\lambda_i)_{i=1}^N$, and all local operations $(\phi_i)_{i=1}^N$, and bipartite d -dimensional pure states $(\eta_i)_{i=1}^N$.

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Theorem (arXiv 1305.1020)

There exists a family of channels with input dimension $2n$ such that

$$\mathcal{C}^{\sqrt{n}}(\mathcal{N}) + t_1 \log n + t_2 \leq \frac{\mathcal{C}^n(\mathcal{N}^{\otimes 2})}{2}$$

Additivity violation

Entangled input can increase the capacity.

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There exist some channels Φ and

$$S_{\min}(\Phi \otimes \bar{\Phi}) < S_{\min}(\Phi) + S_{\min}(\bar{\Phi}).$$

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where $S(\cdot)$ is the von Neumann entropy.

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where $S(\cdot)$ is the von Neumann entropy.

(2) $\bar{\Phi}$ is the complex conjugate of Φ .

(3) The intuition behind is

$$U \otimes \bar{U}|b\rangle = |b\rangle$$

where $|b\rangle$ is a Bell state.

Improvement

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Theorem (Fukuda)

The additivity violation can be proved by Lévy's Lemma and ϵ -net method when $l \sim n \gtrsim k^2$ where l, n, k are dimensions of input, output and environment spaces.

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The additivity violation can be proved by Lévy's Lemma and ϵ -net method when $I \sim n \gtrsim k^2$ where I, n, k are dimensions of input, output and environment spaces.

- (1) Previously, $I \sim n$ case was proven by Aubrun, Szarek and Werner (ASW), but with some stronger constraints on $\frac{I}{k^2}$ and $\frac{n}{k^2}$, which we do not need.
- (2) The same method was used by Hayden, Leung and Winter to show that typical subspaces are highly entangled. Their technique was improved by our new trick.

Is the pair additive?

We know that typically

$$S_{\min}(\Phi \otimes \bar{\Phi}) < S_{\min}(\Phi) + S_{\min}(\bar{\Phi})$$

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But it still may be, that

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because entangled inputs may work well only for the pair $\Phi \otimes \bar{\Phi}$ and may not go beyond.

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Theorem (Fukuda, Nechita)

If you fix an input state and take random channels, then products of Bell states minimize the output entropy for $(\Phi \otimes \bar{\Phi})^{\otimes n}$.

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Theorem (Fukuda, Nechita)

If you fix an input state and take random channels, then products of Bell states minimize the output entropy for $(\Phi \otimes \bar{\Phi})^{\otimes n}$.

This could be stepping stones for the above conjecture.

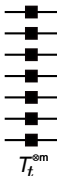
What is the optimal rate of information storage in a quantum memory?

Noise channel $\mathcal{T}_t : \mathfrak{M}_d \rightarrow \mathfrak{M}_d$ is Markovian, i.e. $\mathcal{T}_t = e^{t\mathcal{L}}$.


$$\mathcal{T}_t$$

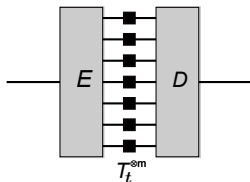
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$$\mathcal{T}_t^{\otimes m}$$

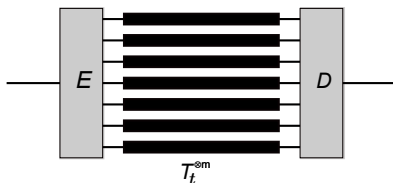
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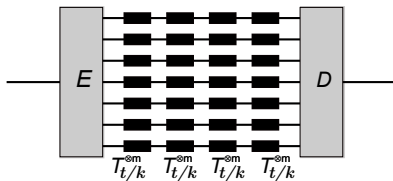
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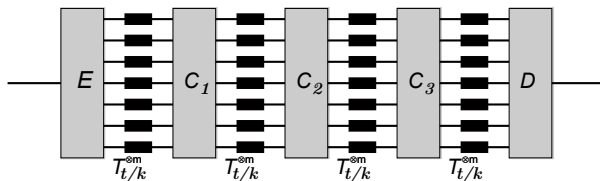
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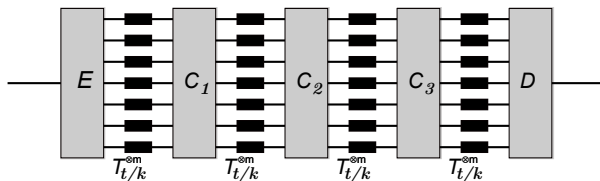
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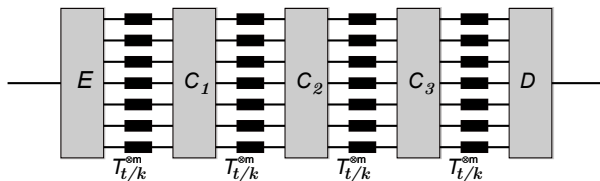
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Quantum subdivision capacity: $\mathcal{Q}_c(t\mathcal{L})$

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Quantum subdivision capacity: $\mathcal{Q}_{\mathcal{C}}(t\mathcal{L})$

Different choices of $\mathcal{C}$ lead to different capacities!

Definition (MH, Reeb, Wolf 2013)

The $\mathfrak{C}$ –**quantum subdivision capacity** of $t\mathcal{L}$ is then defined as the supremum of asymptotical achievable rates

$$Q_{\mathfrak{C}}(t\mathcal{L}) := \sup\{R \in \mathbb{R}^+ : R = \limsup_{\nu \rightarrow \infty} \frac{n_{\nu}}{m_{\nu}}\}.$$

such that the asymptotic communication error vanishes

$$\inf_{k, \mathcal{E}, \mathcal{D}, \mathcal{C}_1, \dots, \mathcal{C}_k} \left\| \text{id}_2^{\otimes n_{\nu}} - \mathcal{D} \circ \prod_{l=1}^k \left(\mathcal{C}_l \circ \left(e^{\frac{t}{k} \mathcal{L}} \right)^{\otimes m_{\nu}} \right) \circ \mathcal{E} \right\|_{\diamond} \rightarrow 0 \quad \text{as } \nu \rightarrow \infty.$$

Infimum goes over:

- ▶ $k \in \mathbb{N}$ number of subdivisions
- ▶ $\mathcal{E} : \mathfrak{M}_2^{\otimes n_{\nu}} \rightarrow \mathfrak{M}_d^{\otimes m_{\nu}}$ and $\mathcal{D} : \mathfrak{M}_d^{\otimes m_{\nu}} \rightarrow \mathfrak{M}_2^{\otimes n_{\nu}}$ quantum channels
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Infinitely divisible coding maps

1. **Example:** Let $\mathfrak{C}$ be the set of **infinitely divisible quantum channels**

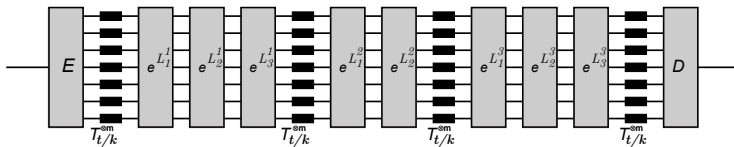
1. **Example:** Let $\mathfrak{C}$ be the set of **infinitely divisible quantum channels**

- $\mathcal{C}_I = \prod_{i=1}^N e^{\mathcal{L}_i^I}$ for some coding Liouvillians $\mathcal{L}_i^I$

Infinitely divisible coding maps

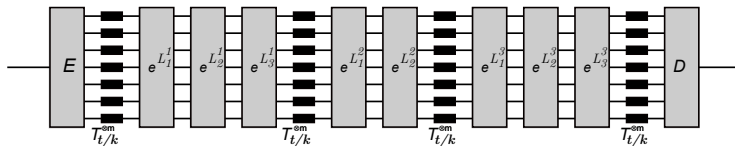
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Theorem (MH, Reeb, Wolf 2013)

For any noise Liouvillian $\mathcal{L} : \mathfrak{M}_d \rightarrow \mathfrak{M}_d$ and any $t \in \mathbb{R}^+$ we have

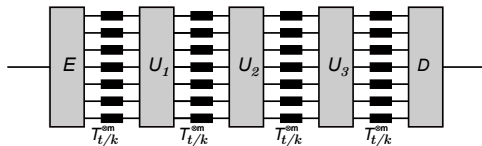
$$\mathcal{Q}_{ID}(t\mathcal{L}) = \log(d)$$

Unitary coding maps

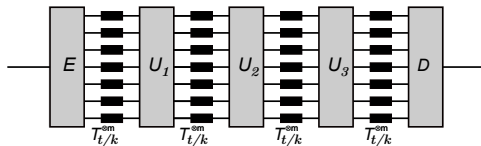
2. **Example:** $\mathcal{C} = \mathcal{U}$, i.e. unitary coding maps.

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Theorem (MH, Reeb, Wolf 2013)

For any noise Liouvillian $\mathcal{L} : \mathfrak{M}_d \rightarrow \mathfrak{M}_d$ and any $t \in \mathbb{R}^+$ we have

$$Q_{\mathfrak{U}}(t\mathcal{L}) > 0$$

Is $Q_{\mathcal{U}}(t\mathcal{L})$ also $\log(d)$?

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Liouvillian depolarizing onto state $\rho_0 \in \mathfrak{M}_d$:

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$\rightsquigarrow$ Generates depolarizing channel:

$$e^{t\mathcal{L}^{\text{dep}}}(\rho) = (1 - e^{-t}) \text{tr}(\rho) \rho_0 + e^{-t} \rho$$

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Theorem (MH, Reeb, Wolf 2013)

For the noise Liouvillian $\mathcal{L}^{\text{dep}} : \mathfrak{M}_d \rightarrow \mathfrak{M}_d$ and $t \in \mathbb{R}^+$ we have

$$\mathcal{Q}_{\mathcal{L}}(t\mathcal{L}^{\text{dep}}) \leq \log(d) - (1 - e^{-t}) S(\rho_0)$$

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as $\nu \rightarrow \infty$. Infimum goes over:

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Definition (MH, Reeb, Wolf 2013)

The $\mathfrak{C}$ –**continuous quantum capacity** of noise Liouvillian $t\mathcal{L}$ is then defined as the supremum of asymptotical achievable rates

$$Q_{\mathfrak{C}}^{\text{cont}}(t\mathcal{L}) := \sup\{R \in \mathbb{R}^+ : R = \limsup_{\nu \rightarrow \infty} \frac{n_{\nu}}{m_{\nu}}\}.$$

such that the asymptotic communication error vanishes

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$$\mathcal{Q}_{\mathfrak{C}}^{\text{cont}}(t\mathcal{L}) := \sup\{R \in \mathbb{R}^+ : R = \limsup_{\nu \rightarrow \infty} \frac{n_{\nu}}{m_{\nu}}\}.$$

such that the asymptotic communication error vanishes

$$\inf_{\mathcal{E}, \mathcal{D}, \mathcal{L}_c} \left\| \text{id}_2^{\otimes n_{\nu}} - \mathcal{D} \circ T \exp \left(\int_0^t \mathcal{L}^{\oplus m_{\nu}} + \mathcal{L}_c(t') dt' \right) \circ \mathcal{E} \right\|_{\diamond} \rightarrow 0.$$

as $\nu \rightarrow \infty$. Infimum goes over:

- ▶ $\mathcal{E} : \mathfrak{M}_2^{\otimes n_{\nu}} \rightarrow \mathfrak{M}_d^{\otimes m_{\nu}}$ and $\mathcal{D} : \mathfrak{M}_d^{\otimes m_{\nu}} \rightarrow \mathfrak{M}_2^{\otimes n_{\nu}}$ quantum channels
- ▶ $\mathcal{L}_c \in \mathfrak{C}$ time-dependent coding Liouvillians from the subset $\mathfrak{C}$

Can dissipation improve quantum capacity?

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Answer: Yes

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Answer: Yes

Even for usual quantum capacity we have:

Theorem (MH, Reeb, Wolf 2013)

There exist time-independent Liouvillians $\mathcal{L} : \mathfrak{M}_d \rightarrow \mathfrak{M}_d$ and $\mathcal{L}' : \mathfrak{M}_d \rightarrow \mathfrak{M}_d$ where $\mathcal{L}'$ is purely dissipative such that

$$\mathcal{Q}(e^{\mathcal{L}}) < \mathcal{Q}(e^{\mathcal{L}+\mathcal{L}'}).$$

Thank you for your attention.

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**For more information regarding CQC and a list of publications
associated with this project please visit:**

<http://www-m5.ma.tum.de/CQC>